Evaluating Risky Prospects If There Are Transaction Costs

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1 Introduction

An approach to differentiate gains and losses has appeared to be productive in describing individual decision making in situations of risk and uncertainty. It was originally introduced by Markowitz (1952) and later on adopted by Kahneman and Tversky as a part of their prospect theory (Kahneman and Tversky (1979); Tversky and Kahneman (1992)). For example, if an individual of wealth 10 prefers gaining 1 for sure over gaining 2 with probability 1/2, he may also, having wealth of 12, prefer loosing 2 with probability 1/2 over loosing 1 for sure. These prospects are equivalent in terms of final wealth, but are different once considered as deviations relative to the current wealth, latter can explain the observed difference in choice.

Commonly, gains and losses are evaluated on their own without taking current wealth into account. Let us call this approach a basic reference point

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approach, where the reference point is usually the current wealth or the future planned wealth. Though this approach works well descriptively, it violates asset integration and it does not explain on its own why people seem to judge in terms of gains and losses rather than in terms of final wealth. This paper also differentiates between gains and losses but it adds a new sense to this differentiation by means of transaction costs. It is shown that in theory transaction costs can explain the observed behaviour while maintaining the principle of asset integration.

A complete theory of choice under risk or uncertainty has several components. In this paper transaction costs address only the evaluation of the outcomes of a prospect, so the following theory is not a complete theory of choice. However, it may be used in any theory of choice that employs a basic reference point approach in place of that what makes it rather general.

To proceed we first discuss what kind of transaction costs I have in mind and next we set up a model that shows how these transaction costs change the evaluation of prospects.

2 Transaction Costs

If a student unexpectedly receives an extra €1000, he can simply put it on his bank account. If he unexpectedly looses €1000, it may happen that he has to rearrange his bank loan or to ask relatives to fulfil this payment. Moreover, it may happen that he has to call his friends to shift their vacation plans to a cheaper alternative.

The example sketches the two types of transaction costs: the costs of arranging the payment itself and the costs of changing the lifestyle, whether

the current one or the planned one.[1] The presence of such transaction costs will imply different behaviour when considering either a gain or a loss. Not accounting for these costs will attribute all the difference to either a gain or a loss per se.

3 Model and Discussion

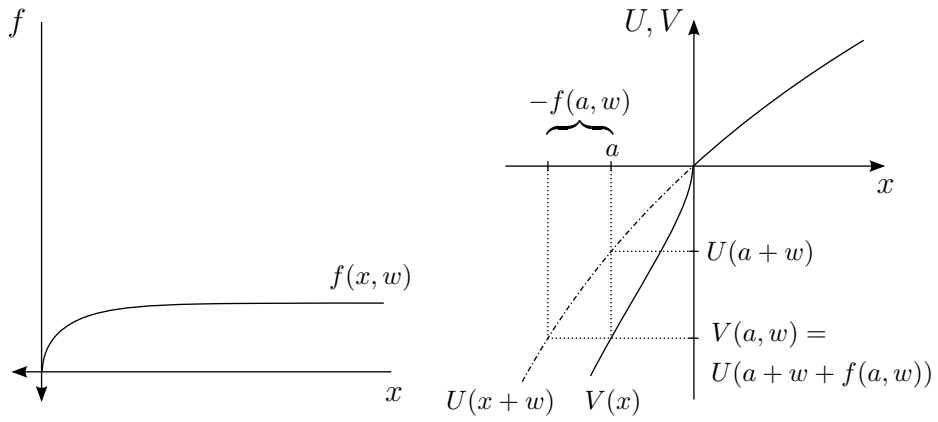
Many economic prospects consist of monetary outcomes. Moreover, transaction costs are usually measured in money. Consequently, this paper focuses on monetary outcomes.[2]

The reference point is chosen to be the current wealth if the prospect is immediate and to be the future planned wealth if it is a delayed prospect. Let $x \in \mathbb{R}$ denote a monetary outcome of a prospect and let x be taken relative to such a reference point. Outcome x may be coded in two different ways: either it includes the associated transaction costs, or it does not. If it does include the transaction costs, let us call it a *gross outcome*. If it does not, let us call it a *net outcome*. If x is a net outcome and if $x > 0$, it is assumed there are no transaction costs. If $x < 0$, the occurring transaction costs are denoted by $f(x, w) < 0$; these costs are allowed to depend upon the current wealth w. This way if an individual of wealth w faces a *gross outcome* x, his final wealth will be $w + x$. If he faces a *net outcome* x, his final wealth will be $w + x + f(x, w)$.

[1] Personal observations suggest that some people also exhibit mental costs for changing the current or the future lifestyle, but what is the true psychological reason for that – whether that is some type of costs indeed or not – is debatable and is not elaborated.

[2] Extending the theory to non-monetary outcomes will involve either defining a proper equivalence with monetary outcomes or defining ordering and operations of summation and negation on a set of outcomes and transaction costs.

Figure 1: Constructing a relative utility function



Let $U(\cdot)$ be a utility function over final wealth. Define

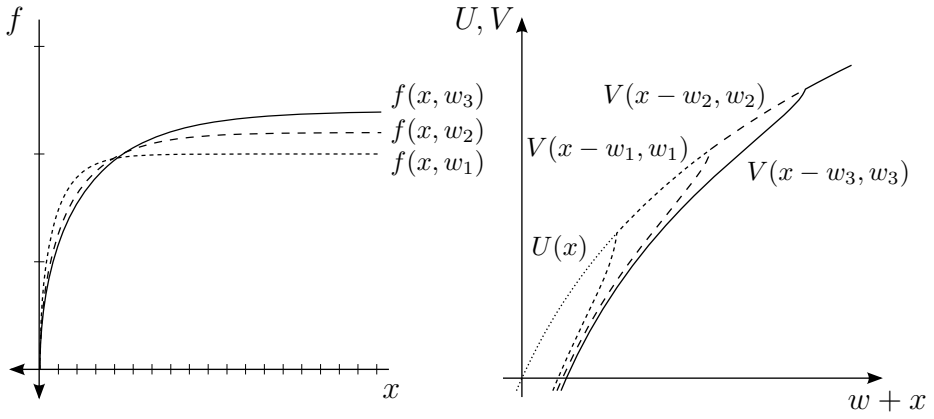
$$V(x,w) = \begin{cases} U(w+x) & \text{if} \quad x \geq 0, \\ U(w+x+f(x,w)) & \text{if} \quad x < 0. \end{cases} \tag{1}$$

With this definition, if the way in which an individual values outcomes is described through a function defined over gains and losses with outcomes not taking the transaction costs into account, this function will be precisely $V(x,w)$. Let us call it a relative utility function.

In the following discussion a continuous f will be addressed first and a discrete case will be considered later on.

Figure 1 presents a typical $U(x)$, strictly increasing and concave in x, a certain function $f(x,w)$ that describes the transaction costs and the resulting $V(x,w)$. There are two stylised facts about what is called $V(x,w)$ here: it is usually steeper for losses, i.e. losses loom larger than gains, and it is usually convex for losses while being concave for gains, i.e. there is risk seeking for losses while there is certainty seeking for gains (see, e.g., Kahneman and Tversky (1979)). The figure illustrates how transaction costs that are steep

Figure 2: Relative utility functions for different initial wealth ($w_3 > w_2 > w_1$)



Note: In the left picture the f-axis is scaled relative to the x-axis as indicated.

at $x = 0$ can explain both findings.[3]

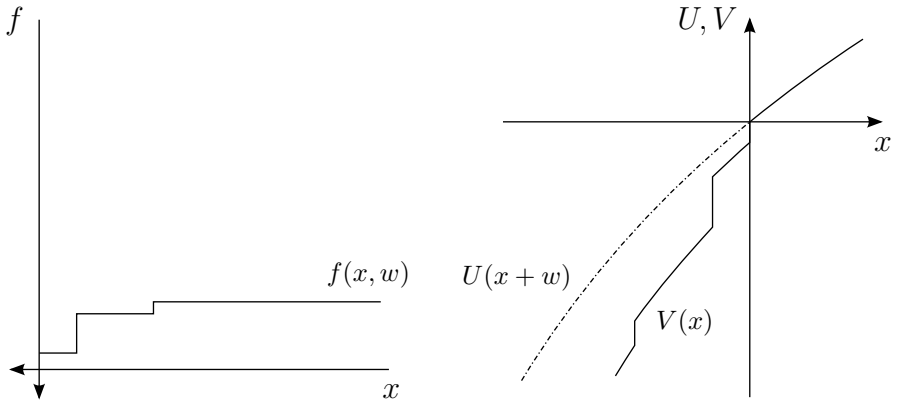
Consider now figure 2. It presents how a relative utility function changes with respect to wealth w. The following was assumed for the specification of the transaction costs. First, a wealthier individual has lower transaction costs given the same loss if the loss is not too big because he does not have to adjust his current or planned lifestyle that much. Secondly, if the loss is big enough, a wealthier individual will have higher transaction costs since he is assumed to be engaged into more economic activities that he will have to rearrange now.[4]

The figure gives a hint on how to distinguish empirically the reference point approach from the transaction costs approach. From one side, they are alike, though not equivalent. Given $U(x)$ and $f(x, w)$, one immediately

[3]However, if the cumulative transaction costs stop increasing at some point and if $U(x)$ is concave everywhere, $V(x, w)$ will turn to be concave again for large losses, like it happens in the figure.

[4]This is not a definitive proposal for the form of the transaction costs function, rather an example.

Figure 3: Discrete transaction costs



comes up with $V(x, w)$. Given $U(x)$ and given $V(x, w)$ for some fixed w, one can often find the corresponding $f(x, w)$, though not always as $f(x, w)$ is apparently restricted to be nondecreasing in x. From the other side, the basic reference point approach does not give any specific predictions on how $V(x, w)$ is going to change if w changes. While under plausible assumptions about the transaction costs our model will predict a certain pattern of how $V(x, w)$ changes with respect to w, e.g. the one in the figure. If such a pattern holds empirically we can not reject either approach but should prefer the transaction costs approach as it explicitly explains the pattern. If such a pattern does not hold empirically, we can reject the transaction costs approach.

Transaction costs can be reasoned to change discretely rather than continuously with respect to the size of the loss, e.g. once the loss crosses the monthly limit on a credit card, etc. Figure 3 illustrates the case.

Payne (2005) gives experimental evidence that overall probabilities of a gain and a loss play a role in the evaluation of prospects. Diecidue and van de Ven (2006) show that accounting for these overall probabilities is equivalent

to $V(x, w)$ that is discontinuous at $x = 0$. Figure 3 futher shows that this discontinuity of $V(x, w)$ is equivalent to the presence of transaction costs that make a positive jump when a gain changes into a loss. Hence, discrete transaction costs can provide a rationale why the overall probabilities of a gain and a loss are significant.

4 Conclusions

We have seen how transaction costs can be an add-on story to the basic reference point approach. It was shown that the presence of transaction costs can explain why looses loom larger than gains, why there is risk seeking for losses while there is certainty seeking for gains and why the overall probabilities of a gain and a loss can be significant.

Comparing with the basic reference point approach, the presented approach is consistent with asset integration. Moreover, transaction costs are subject to economic intuition, i.e. it can be reasoned what shapes of $f(x, w)$ are more or less plausible in a given context, therefore the presented approach has a potential for more specific predictions. How relevant these advantages are however depends upon how much of reference dependence is accounted for with such transaction costs. This is an empirical question and it was not our topic, rather it can be a topic for future research.

References

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