

Targeted Competition: Choosing Your Enemies in Multiplayer Games

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Abstract

We consider a dynamic (differential) game with three players competing against each other. Each period each player can allocate his resources so as to direct his competition towards particular rivals – we call such competition targeted. We show that if the players are myopic, the weaker players eventually lose the game to their strongest rival. If instead the players are sufficiently non-myopic, then each player concentrates more on fighting his strongest opponent. Consequently, the weaker players grow stronger, the strongest player grows weaker and eventually all the players converge and stay in the game. Targeted competition setting can be applied to a wide variety of cases: competition between firms, competition between political parties, warfare.

Key Words: targeted competition, dynamic oligopolies, differential games.

JEL Classification: C73, D43.

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1 Introduction

Competition lies at the heart of economics and has been studied extensively. However, there is a class of competition mechanisms that is abundant in practice but which, to the best of our knowledge, has not yet been studied specifically in the literature – those are mechanisms providing a competitor with an ability to target his rivals on an individual basis. We group such mechanisms under the common label of targeted competition. The few examples that follow illustrate how pervasive targeted competition is. On product markets, firms may decide to develop a product that is closer along one characteristic to that of a particular competitor. A multinational corporation may decide to invest relatively more in a market shared with a particular rival (see, for example, surveys by Lancaster, 1990; Gabszewicz and Thisse, 1992; Bailey and Friedlaender, 1982). Another example of targeted competition is comparative advertisement (see, for example, Barigozzi and Peitz, 2007; Anderson et al., 2009), a practice of running ads that directly compare one’s products to that of the rivals. Unethical practices, for example launching fabricated lawsuits against specific rivals, provide further ways to target competitors. Targeted competition is not restricted to economics only. Think about the ways political parties and politicians compete through their support for specific programs, or how different governments try to protect local industries through trade barriers. Finally, a warfare stays as an ultimate example of targeted competition.

Targeted competition includes a strategic consideration that does not arise in non-targeted competition: a player (a firm, a political party, an army) can influence the balance of powers among his rivals by choosing whom to compete against; in turn, that determines how much this player wins or loses competing with those rivals in the periods to come. In particular, one may intuitively expect the weaker players to direct more resources towards fighting the strongest player rather than fighting each other. Indeed, otherwise the strongest player stands a good chance of forcing the weaker ones out of the game (as time goes by).

Any model of targeted competition should have the following two characteristics: 1) there should be three players or more – otherwise the competition cannot be targeted; and 2) the analysis should be dynamic – the aforementioned strategic consideration can be only studied in a dynamic setting. The closest matching strand of the literature then is that of dynamic oligopoly models. Though many scenarios of dynamic competition are studied (in-

ventories (Kirman and Sobel, 1974), sticky prices (Fershtman and Kamien, 1987), evolution of sales (Dockner and Jørgensen, 1988), varying profit opportunities (Ericson and Pakes, 1995), collusive behaviour (Fershtman and Pakes, 2000), etc.), targeted competition is not part of the analysis. This paper aims to be a first step towards filling this gap.

We develop a model of targeted competition that does not focus on case-specific aspects of competition but rather focuses on the general ability to target selected rivals. Each player in the model is characterised by his relative power – the amount of resources this player has. The power of a player can be distributed to fight each of the player’s rivals. We first show that myopic players prefer to fight more with their weakest opponent. Consequently, the strongest player grows in power and eventually outcompetes the weaker players. Vice versa, we show that if players are non-myopic and do not discount future payoffs too much, then the weaker players concentrate more on fighting their strongest opponent (provided no player is too strong to start with). Consequently, the strongest player becomes weaker over time and all the players converge in power to a common level and survive.

So, if a competition on a certain market is targeted and the competitors are forward-looking, then this competition is sustainable. On the other hand, if there are no ways to target particular rivals, then the market becomes a monopoly. From a practical perspective, this result is relevant for policies that influence the effectiveness of targeted competition. Some recent examples of such policy questions are: whether or not to allow comparative advertisement (Barigozzi and Peitz, 2007); whether to legislate network neutrality – network neutrality prohibits internet providers to differentiate their traffic in any way, including price differentiation (Economides and Tåg, 2009; Kocsis and de Bijl, 2007).

It is tempting to view the fact that the weaker players fight together against their strongest rival as a form of tacit collusion. It is, however, conceptually different. Collusive behaviour in repeated games is sustained by the credible threat that other players will punish the one who deviates from the equilibrium. In our game the equilibrium concept is Markov perfect equilibrium, hence the strategies do not depend upon past actions and so there are no strategies with retrospective punishment. In our case it is the dynamic structure of the game that pushes the weaker players to fight together for the common cause: if they are to prefer fighting each other for the sake of immediate gains, then the power of the strongest player will grow up to the point at which, eventually, he can outcompete his rivals. If this threat

of losing the game is large enough, then the weaker players will fight more against the strongest player and their behaviour will be alike to that of tacit collusion.

There are two related games that have been studied in the literature: colonel Blotto games (see, e.g., Roberson, 2006) and truel games (Kilgour, 1971).

A colonel Blotto game is a game between two players that share several battlefields. Each player divides his army between the battlefields, a battlefield is won by the larger force, a player who wins more battlefields wins the game. The game of targeted competition that we study can be viewed as a game of three players and three battlefields, where each pair of players share a battlefield and where there is no battlefield that is shared by all the players. Then the similarity of our game to colonel Blotto games is the ability of the players to choose how to split their powers against their opponents. The main differences are: 1) there are three players in our game, 2) our game is dynamic – the winner is not determined at once, rather the winner of this round becomes stronger and the game continues.

A truel game is an extension of a duel game. There are three players, each with a gun. Each round each player chooses whom to shoot and kills his opponent with a certain chance that depends upon his skill; if two or more players are still alive the game continues. Like in our game, there is a choice of the opponent, there are dynamics and there is a consideration that killing a certain player influences your chance of survival in the rounds to come. The main differences are: 1) in our game the payoff of the game is a discounted sum of the payoffs in each round, so each round is valuable, whereas in a truel game the payoff is 1 if the player survives and 0 otherwise; 2) in our game if the player is “shot”, he does not die at once but rather becomes relatively weaker; 3) in a truel game a player chooses to fight either one opponent or the other, whereas in our game a player chooses *how much* to fight one opponent and *how much* to fight the other (a continuous choice).

So, our game has structural similarities to those of colonel Blotto and truel games, but we think the named differences make our model more appropriate for the aforementioned examples of targeted competition.

We use a linear-quadratic specification for our model. Among the types of differential games that tend to have analytical solutions (see Dockner et al., 2000), a linear-quadratic type is the only one that satisfies our assumptions on payoffs (diminishing marginal returns, etc). So, while restrictive, it is our only choice if we want to present a model that is analytically tractable. We

discuss this point in greater detail in the following section.

The rest of the paper is organised as follows. The next section presents the model, which is inspired by the above examples about targeted competition between firms. Section three considers the simple case of myopic players and shows that only the strongest player survives as time goes by. In section four, we show that if players are not myopic, the discount rate is sufficiently small and if no player is too strong to start with, then there is an equilibrium where all the players converge in power and remain in the game. The last section concludes.

2 Setup

There are three players, 1, 2, and 3 – firms, political parties, armies, etc. The players are involved in a dynamic competitive game. Each player i at time $t \in [0, \infty)$ is characterised by a state variable $x_i(t)$ being the amount of resources he can use in competition with his rivals at time t . We call this variable the “power” of player i . It can be the market share of a firm, the amount of personnel the firm has, how large and how good its credit resources are or how well the managers are connected; it can be the electoral base or the number of seats in parliament; it can be the number of military units.

For convenience, let $x = (x_1, x_2, x_3)$. At any time t the powers of the players, $x(t)$, are common knowledge.

The initial state $x(0)$ is normalised so that $\sum_i x_i(0) = 1$ (later on we will see that $\sum_i x_i(t) = 1$ for any t) and also no player is too strong to start with. Formally, $x(0) \in X$, where

$$X = \left\{ x \in \mathbb{R}^3 \mid \sum_i x_i = 1, x_i < \frac{2}{5} \forall i \right\}$$

The reason for the restriction $x_i(0) < \frac{2}{5}$ is a technical one. Under this restriction the best responses (that we are to analyse later on) attain inner solutions and the whole problem is tractable analytically. If one considers a more natural restriction that $x_i(0) < \frac{1}{2}$, then a numerical solution for a system of differential equations shall be exercised. Not to overcomplicate the exposition of our ideas we circumvent the difficulty of solving the problem numerically by considering a smaller region for x .

In the following analysis we focus on Markov strategies: a strategy of any player depends only on the current state x and does not depend on the past actions of the players. Our choice for Markov strategies comes from the objective of the paper – to study whether forward looking behaviour can produce collusive type outcomes without invoking the usual means of sustaining collusion (such as trigger strategies). Moreover, considering Markov strategies has appealing properties. First, an equilibrium in Markov strategies is also an equilibrium in a game with non-Markov strategies. Second, suppose a game with general strategies has multiple equilibria and one of them is a Markov equilibrium. One way to select an equilibrium is to explore whether there is a focal point (Schelling, 1960). If simplicity makes a focal point, then the Markov equilibrium is selected. There are also other reasons, both theoretical and practical, for opting for Markov strategies – see the introduction in Maskin and Tirole (2001).

A player can target his rivals, i.e. a player can decide how much he wants to fight each of his opponents. y_{ij} denotes the amount of power player i uses to fight against player j . Let $y_1 = (y_{12}, y_{13})$, $y_2 = (y_{21}, y_{23})$, $y_3 = (y_{31}, y_{32})$ and $y = (y_1, y_2, y_3)$.

As we consider Markov strategies, the actions of the players are conditioned upon the state of the game, and so y_{ij} are functions of x .

Each player uses all his power to fight his opponents¹ and what amount he uses can not be negative, therefore

$$Y_i(x) = \left\{ y_i \mid y_{ij} \geq 0, \sum_j y_{ij} = x_i \right\} \quad (1)$$

Contemplating targeted competition brings forward two effects. First, players have immediate gains from “fighting”, e.g. profits in case of firms, or utilities of top managers; political contributions in case of political parties; access to natural resources in case of warfare for economic reasons. We refer to these gains as instantaneous payoffs. Second, if for some time a player is opposed to another player with less power, then the former player becomes even stronger while his opponents becomes weaker. For example, if a company invests more in a market than its competitors do, or if a political party supports a certain program more than its rivals do, then the corresponding

¹In our model there are no alternative costs associated with fighting, therefore it is always optimal to use all one’s power for fighting.

customer or electoral base increases relative to that of the rivals. We refer to such dynamics as power shift.

The instantaneous payoff for player i when he is fighting player j is given by $\varphi(y_{ij}, y_{ji})$, with 1) $\varphi(0, y_{ji}) = 0$, which happens if player i doesn't fight; 2) $\varphi(y_{ij}, y_{ji})$ strictly increasing in y_{ij} and, for $y_{ij} > 0$, strictly decreasing in y_{ji} ; 3) $\varphi(y_{ij}, y_{ji})$ strictly concave in y_{ij} (decreasing marginal returns).

Dockner et al. (2000) identify three types of differential games that admit analytical solutions: linear-quadratic, linear state and exponential games. Among those, only linear-quadratic games can exhibit payoffs satisfying the aforementioned assumptions². So, we take a quadratic specification for φ :

$$\varphi(y_{ij}, y_{ji}) = (a - b_1 y_{ij} - b_2 y_{ji}) y_{ij}$$

where $b_1 > 0$, $b_2 > 0$ and $a \geq 2b_1 + b_2$. This is the most general quadratic specification that would satisfy our assumptions on the relevant domain ($0 \leq y_{ij} \leq 1$, $0 \leq y_{ji} \leq 1$). Further, for simplicity, we assume that $b_1 = b_2 = b$, so

$$\varphi(y_{ij}, y_{ji}) = (a - b(y_{ij} + y_{ji})) y_{ij}$$

where $b > 0$ and $a \geq 3b$. Let us note that if y_{ij} is interpreted as output, then $\varphi(y_{ij}, y_{ji})$ can be interpreted as the profit of a firm in a Cournot duopoly game with linear demand.

Let $\pi_i(y)$ denote the sum of all the instantaneous payoffs that player i receives from fighting his opponents. We have

$$\pi_i(y) = \sum_{j \neq i} \varphi(y_{ij}, y_{ji})$$

Power does not enter the instantaneous payoff function per se. However, becoming more powerful will yield higher payoffs as more power can be used to compete against rivals, thus improving the outcomes of future competition rounds.

If player i fights player j harder than player j fights player i ($y_{ij} > y_{ji}$), then player i becomes more powerful, while player j becomes less powerful. These power shift dynamics are assumed to be linear in y :

²Solvability of linear-quadratic games makes them a popular tool for analysis of dynamic oligopolies – see, e.g. Fershtman and Kamien (1987), Cellini and Lambertini (1998).

$$\begin{aligned}\dot{x}_i(t) &= f_i(y(x(t))) \\ f_i(y) &= \sum_{j \neq i} (y_{ij} - y_{ji}) k\end{aligned}\tag{2}$$

where $k > 0$ stands for the power shift intensity.

We note here that from $\sum_i x_i(0) = 1$ and from (2) it follows that $\sum_i x_i(t) = 1$ for all t .

If $x(t)$ reaches the boundary of X , the game ends. T denotes the ending time. Formally,

$$T = \inf\{t \geq 0 \mid x(t) \notin X\}$$

If the game never ends we slightly abuse notation and write $T = \infty$.

If the game ends, each player i receives a terminal payoff S_i , the strongest player wins, the weaker players lose:

$$S_i(x) = \begin{cases} M & \text{if } x_i > x_j \ \forall j \neq i \\ 0 & \text{otherwise} \end{cases}$$

where $M > 0$.³

The rationale for ending the game if the boundary of X is approached is as follows. If one of the players becomes sufficiently strong, it is reasonable to expect him to eventually outcompete his rivals. To simplify the game we stop it at this time and assign a strictly positive payoff of M to the strongest player and a zero payoff to the weaker players. As we will see later on, the results do not depend upon the size of M . Yet it is helpful to think of it as of a payoff that is higher than what the strongest player could have got if he was to continue the competition. Losing, on the other hand, means that a player quits the game (a firm loses its markets, etc) and the stream of the instantaneous payoffs ends – so losing yields zero payoff.

The payoff for the whole game is the discounted stream of the instantaneous payoffs plus the discounted terminal payoff, so the payoff for player i is

$$U_i = \int_0^T e^{-rt} \pi_i(y(x(t))) dt + e^{-rT} S_i(x(T))\tag{3}$$

where r is a discount rate.

³If the game ends and two players are equally strong, they both lose. This assumption is made for simplicity and does not change the results.

So, our setup is a differential game with simultaneous play (see Dockner et al., 2000) and we restrict our attention to Markov strategies. The strategies are functions $y(x)$ satisfying (1), the state variables x evolve according to (2) and the objective functions are given by (3).

3 Analysis

We consider two cases: a case with myopic players and a general case. In both cases we look for Markov perfect equilibria (MPE) and analyse the resulting equilibrium dynamics.

In what follows we denote the best response strategies with \tilde{y} and the equilibrium strategies with \hat{y} .

3.1 Myopic Players

The players are myopic if they only focus on the current gains. For a myopic player i the payoff of the game at time t is

$$U_i(t) = \pi_i(y(x(t)))$$

The dynamics of the myopic case are summarised by the following proposition (we limit our attention to a general initial state, when one of the players is strictly stronger than the rest).

Proposition 1. *Suppose, without a loss of generality, that $x_1(0) > x_2(0)$, $x_1(0) > x_3(0)$. Then there exists a unique MPE. Moreover, the equilibrium dynamics are such that the game ends and the strongest player wins, i.e. $T < \infty$ and $x_1(T) > x_2(T)$, $x_1(T) > x_3(T)$*

Proof. Maximising $U_i(t)$ in (y_{ij}, y_{ik}) w.r.t. $y_{ij} + y_{ik} = x_i$ gives a unique best response

$$\tilde{y}_{ij}(x) = \frac{x_i}{2} + \frac{y_{ki}(x) - y_{ji}(x)}{4}$$

(a boundary solution is also possible but it is straightforward to check that it is never attained for $x \in X$).

Given the above best response functions we can solve for a unique equilibrium point. We get

$$\hat{y}_{ij}(x) = \frac{x_i}{2} + \frac{x_k - x_j}{10} \tag{4}$$

As we are considering Markov strategies, (4) constitutes a unique Markov perfect equilibrium.

Plugging (4) into (2) and using $x_1 + x_2 + x_3 = 1$ gives

$$\dot{x}_i(t) = \frac{9k}{5} \left(x_i(t) - \frac{1}{3} \right)$$

Therefore

$$\dot{x}_1(t) - \dot{x}_i(t) = \frac{9k}{5} (x_1(t) - x_i(t)) \quad (5)$$

X is bounded and $x_1(0) > x_i(0)$ for $i \in \{2, 3\}$. It then follows from (5) that $x(t)$ reaches the boundary of X at some time T and that $x_1(T) > x_i(T)$ for $i \in \{2, 3\}$. \square

This case illustrates the intuition that if the players are myopic and only pursue their instantaneous payoffs then they have no incentives to fight more against the stronger player. As a consequence, the weaker players lose.

3.2 Forward-looking Players

If the players are myopic, then the weaker players lose in the equilibrium. The question is, if the players are sufficiently non myopic, i.e. if r is sufficiently small so that the players value their future profits high enough, will it be the case the dynamics are reversed? We give a positive answer to this question.

Proposition 2. *If $r < \frac{4k}{3}$, then there exists an MPE such that for all i $x_i(t) \rightarrow \frac{1}{3}$ as $t \rightarrow \infty$.*

Proof. We prove the proposition by construction: we state an equilibrium candidate possessing the property that $x_i(t) \rightarrow \frac{1}{3}$ and then check that it is an equilibrium indeed. Let

$$\hat{y}_{ij}(x) = \frac{x_i + c(x_k - x_j)}{2} \quad (6)$$

$$c = \frac{1}{18} \left(5\frac{r}{k} - 14 - \sqrt{\left(25\frac{r}{k} - 76 \right) \left(\frac{r}{k} - 4 \right)} \right) \quad (7)$$

From $\sum_i x_i(t) = 1$, from (2) and from (6) it follows that

$$\dot{x}_i(t) = \frac{3k(c+1)}{2} \left(x_i(t) - \frac{1}{3} \right) \quad (8)$$

If $r < \frac{4k}{3}$, then from (7) it follows that $c < -1$. Consequently, from (8) it follows that $x_i(t) \rightarrow \frac{1}{3}$ as $t \rightarrow \infty$.

Let us now prove that (6) constitute an MPE. To do so we need to show that \hat{y}_i is a best response to \hat{y}_j and \hat{y}_k . So, we fix the strategies of players j and k at \hat{y}_j and \hat{y}_k and we consider different strategies of player i . Given the strategies of players j and k , all the possible strategies of player i can be divided into two classes: those strategies that never end the game ($T = \infty$) – let it be class \mathcal{A} , and those that eventually do ($T < \infty$) – class \mathcal{B} . We proceed as follows. First, we restrict the strategies of player i to class \mathcal{A} and show that in this class the strategy \hat{y}_i , as given by (6), is indeed a best response strategy. Second, we extend this result to $\mathcal{A} \cup \mathcal{B}$.

So, let the strategies of player i be restricted to class \mathcal{A} . Let us compute the value function V of player i if every player follows strategy \hat{y} and if the game starts at $x(0) = x$. Solving (8) gives

$$x_i(t) = \left(x_i - \frac{1}{3}\right) e^{3k(c+1)/2 \cdot t} + \frac{1}{3}$$

Therefore (also using $x_1 + x_2 + x_3 = 1$) we have⁴

$$V_i(x) = \int_0^\infty e^{-rt} \pi_i(\hat{y}(x(t))) dt = c_1 \left(x_i - \frac{1}{3}\right)^2 + c_2 \left(x_i - \frac{1}{3}\right) + c_3 + c_4 (x_k - x_j)^2 \quad (9)$$

where

$$\left\{ \begin{array}{l} c_1 = \frac{b(3c-1)}{4(r-3k(c+1))} \\ c_2 = \frac{12a+b(3c-5)}{6(2r-3k(c+1))} \\ c_3 = \frac{3a-b}{9r} \\ c_4 = -\frac{bc(3c-1)}{4(r-3k(c+1))} \end{array} \right. \quad (10)$$

⁴See the appendix for the details of the derivation.

Consider now the Hamilton-Jacobi-Bellman equations:

$$\hat{y}_i(x) \in \text{Arg} \max_{y_i \in Y_i(x)} \left(\pi_i(y_i, \hat{y}_{-i}(x)) + \sum_j \frac{\partial V_i(x)}{\partial x_j} f_j(y_i, \hat{y}_{-i}(x)) \right) \quad (11)$$

$$rV_i(x) = \pi_i(\hat{y}(x)) + \sum_j \frac{\partial V_i(x)}{\partial x_j} f_j(\hat{y}(x)) \quad (12)$$

If these equations are satisfied for all $x \in X$, then \hat{y}_i is a best response to \hat{y}_{-i} (when the strategies of player i are limited to class \mathcal{A} , so that $x(t)$ never leaves X) – see Dockner et al. (2000, chapters 3 and 4).

Equation (12) is automatically satisfied by the way V is constructed. We now check equation (11). Let

$$g(y_i, x) = \pi_i(y_i, \hat{y}_{-i}(x)) + \sum_j \frac{\partial V_i(x)}{\partial x_j} f_j(y_i, \hat{y}_{-i}(x))$$

Using (6), (9) and the definitions for π_i, f_i to expand $g(y_i, x)$ and maximising the result w.r.t. $y_{ij} + y_{ik} = x_i$ gives

$$\tilde{y}_{ij}(x) = \frac{x_i + d(x_k - x_j)}{2} \quad (13)$$

$$d = \frac{1-c}{4} - \frac{ck(3c-1)}{2(r-3k(c+1))} \quad (14)$$

Strategy \hat{y}_i is a best response strategy if (6) coincides with (13), i.e. if $c = d$. Using (14) to expand an equation $c = d$ and simplifying gives

$$18c^2 + \left(28 - 10\frac{r}{k}\right)c + \left(2\frac{r}{k} - 6\right) = 0$$

It is straightforward to check that c as defined in (7) is a solution to the above equation. Hence $c = d$ and \hat{y}_i is a best response.

In principle, it is possible that a corner solution is obtained when maximising $g(y_i, x)$, however it is never the case for $x \in X$.

Consider now an arbitrary strategy $\dot{y}_i(x) \in \mathcal{B}$. With a class \mathcal{B} strategy the game ends at some T (that is determined by $y_i(x)$). Let

$$y_i^n(x, t) = \begin{cases} \dot{y}_i(x) & \text{if } t \leq T - \epsilon_n \\ \hat{y}_i(x) & \text{if } t > T - \epsilon_n \end{cases}$$

where ϵ_n is a sequence, $\epsilon_n > 0$ and $\lim_{n \rightarrow \infty} \epsilon_n = 0$. This strategy $y_i^n(x, t)$ belongs to \mathcal{A} , therefore it gives the same or a lower payoff than the best response strategy $\hat{y}_i(x)$, i.e.

$$\int_0^\infty e^{-rt} \pi_i(\hat{y}(x(t))) dt \geq \int_0^\infty e^{-rt} \pi_i(y^n(x(t))) dt = \int_0^{T-\epsilon_n} e^{-rt} \pi_i(\hat{y}(x(t))) dt + \int_{T-\epsilon_n}^\infty e^{-rt} \pi_i(\hat{y}(x(t))) dt$$

Taking the limit as $n \rightarrow \infty$ gives

$$\int_0^\infty e^{-rt} \pi_i(\hat{y}(x(t))) dt \geq \int_0^T e^{-rt} \pi_i(\hat{y}(x(t))) dt + V_i(x(T))$$

On the other hand, the payoff from employing strategy $\hat{y}_i(x)$ is

$$\int_0^T e^{-rt} \pi_i(\hat{y}(x(t))) dt + S_i(x(T))$$

Therefore, if $S_i(x(T)) \leq V_i(x(T))$, then \hat{y}_i is the optimal strategy in class $\mathcal{A} \cup \mathcal{B}$ as well.

As $x(0) \in X$, then from the definition of X it follows that $x_i(0) < \frac{2}{5}$. Whatever the strategy $\hat{y}(x)$ is, from (2), from (6) and from $x_1 + x_2 + x_3 = 1$ it follows that

$$\dot{x}_i(t) \leq \frac{3k(c+1)}{2} \left(x_i(t) - \frac{1}{3} \right)$$

Consequently, $x(T) < \frac{2}{5}$. At the same time, $x(T)$ belongs to the boundary of X . So, if it was true that $x_i(T) > x_j(T)$ for all $j \neq i$, then it should have been that $x_i(T) = \frac{2}{5}$. As it is not, we have that $x_i(T) \leq x_j(T)$ for at least some $j \neq i$. Therefore $S_i(x(T)) = 0$. But from $\varphi(\hat{y}_{ij}(x), \hat{y}_{ji}(x)) > 0$ it follows that $V_i(x(T)) > 0$.

So, $S_i(x(T)) \leq V_i(x(T))$ and $\hat{y}_i(x)$ is a best response strategy when all possible strategies are considered (class $\mathcal{A} \cup \mathcal{B}$).

In words, a weaker player can choose a strategy to reach the boundary of X , but doing so is not optimal. As for the strongest player, he may prefer to reach the boundary if he is still the strongest player when he does so, but he cannot achieve such dynamics if his rivals are playing the equilibrium strategies. \square

So, for a sufficiently small r there is an equilibrium such that the strongest player declines in his power while the weaker players improve in their powers. Consequently, all the players converge. A notable property of this equilibrium is that each player fights his strongest opponent more.

4 Concluding Remarks

Stackelberg (1952) has argued that a duopoly will never achieve an equilibrium in price/quantity setting strategies. Moreover, the duopolists will engage into fighting for leadership and, consequently, one of them will become predominantly stronger in economic terms, or they will find it beneficial to collude.

“Duopoly is an unstable market form not only in the sense that price is apt to be indeterminate, but much more because it is unlikely to remain as a market form for any length of time. The inherent contradictions in the duopolistic situation press for a solution through the adoption of another market form – monopoly”

We do not say a market of three will attain an equilibrium in prices or quantities. Such strategic variables may as well stay indeterminate. Rather we consider the relative powers of the players. We show that if the three players are sufficiently forward looking and if there are ways for them to target their rivals, then everyone competes more against his stronger rival. Consequently the players converge in their power, and oligopolistic competition is sustainable – it does not boil down to a monopoly.

We have analysed but a basic setup of targeted competition and two possible extensions are worth mentioning – stochastic dynamics and multiple players. Arguably, both extensions would bring the model closer to judging real life situations as outcomes of competition are scarcely deterministic and many examples we talked about (e.g., multiproduct firms) often involve more than three players. The main question here will stay the same: given stochastic dynamics or given multiple (more than three) players in the game will it be more difficult or more easy for the weaker rivals to tacitly coordinate against the strongest one?

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Appendix

Here we give a detailed derivation of (9), (10).

Let $z_i = x_i - \frac{1}{3}$. As $x_1 + x_2 + x_3 = 1$, so $z_1 + z_2 + z_3 = 0$. Next we derive $\pi_i(\hat{y}(z))$.

First,

$$\hat{y}_{ij}(x) = \frac{x_i + c(x_k - x_j)}{2} = \frac{z_i + c(z_k - z_j)}{2} + \frac{1}{6}$$

Then (using $\sum_i z_i = 0$ where appropriate)

$$\begin{aligned}
\pi_i(\hat{y}(z)) &= (a - b(\hat{y}_{ij} + \hat{y}_{ji}))\hat{y}_{ij} + (a - b(\hat{y}_{ik} + \hat{y}_{ki}))\hat{y}_{ik} = \\
&\left(a - b \left(\frac{z_i + c(z_k - z_j)}{2} + \frac{z_j + c(z_k - z_i)}{2} + \frac{1}{3} \right) \right) \cdot \\
&\quad \left(\frac{z_i + c(z_k - z_j)}{2} + \frac{1}{6} \right) + \\
&\left(a - b \left(\frac{z_i + c(z_j - z_k)}{2} + \frac{z_k + c(z_j - z_i)}{2} + \frac{1}{3} \right) \right) \cdot \\
&\quad \left(\frac{z_i + c(z_j - z_k)}{2} + \frac{1}{6} \right) = \\
&\left(a - \frac{b}{3} \right) \left(z_i + \frac{1}{3} \right) - \frac{b(3c-1)}{2} \left(z_k \left(\frac{z_i + c(z_k - z_j)}{2} + \frac{1}{6} \right) + \right. \\
&\quad \left. z_j \left(\frac{z_i + c(z_j - z_k)}{2} + \frac{1}{6} \right) \right) = \\
&\frac{b(3c-1)}{4} z_i^2 + \frac{12a + b(3c-5)}{12} z_i + \frac{3a-b}{9} - \frac{bc(3c-1)}{4} (z_k - z_j)^2
\end{aligned}$$

Let $m = 3k(c+1)/2$, then $z_i(t) = z_i e^{mt}$. So,

$$\begin{aligned}
V_i(z) &= \int_0^\infty e^{-rt} \pi_i(\hat{y}(z(t))) dt = \\
&\int_0^\infty e^{-rt} \left(\frac{b(3c-1)}{4} (z_i e^{mt})^2 + \frac{12a + b(3c-5)}{12} z_i e^{mt} + \right. \\
&\quad \left. \frac{3a-b}{9} - \frac{bc(3c-1)}{4} (z_k e^{mt} - z_j e^{mt})^2 \right) dt = \\
&\frac{b(3c-1)}{4} \frac{1}{r-2m} z_i^2 + \frac{12a + b(3c-5)}{12} \frac{1}{r-m} z_i + \\
&\quad \frac{3a-b}{9} \frac{1}{r} - \frac{bc(3c-1)}{4} \frac{1}{r-2m} (z_k - z_j)^2
\end{aligned}$$

Plugging in $z_i = x_i - \frac{1}{3}$ and $m = 3k(c+1)/2$ gives precisely (9) and (10).